

AS Level Further Mathematics A Y533 Mechanics Sample Question Paper

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

OCR supplied materials:

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You must have:

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A
- Scientific or graphical calculator



INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

- 1 A roundabout in a playground can be modeled as a horizontal circular platform with centre O . The roundabout is free to rotate about a vertical axis through O . A child sits without slipping on the roundabout at a horizontal distance of 1.5 m from O and completes one revolution in 2.4 seconds.

(i) Calculate the speed of the child. [3]

(ii) Find the magnitude and direction of the acceleration of the child. [3]

$$i. \frac{2\pi}{\omega} = t$$

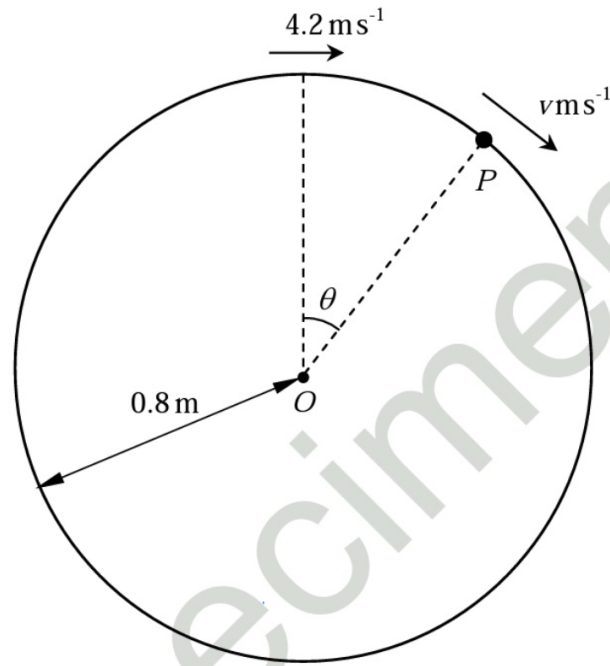
$$\frac{2\pi}{\omega} = 2.4$$

$$\omega = 2.618$$

$$v = r\omega = 1.5 \times 2.618 = 3.93 \text{ ms}^{-1} \text{ (3sf)}$$

$$c. a = \frac{v^2}{r} = \frac{\left(\frac{5}{4}\pi\right)^2}{1.5} = 10.3 \text{ ms}^{-2} \text{ (3sf)}$$

Direction is towards the centre of the roundabout.
(towards centre O).



A smooth wire is shaped into a circle of centre O and radius 0.8 m. The wire is fixed in a vertical plane. A small bead P of mass 0.03 kg is threaded on the wire and is projected along the wire from the highest point with a speed of 4.2 m s^{-1} . When OP makes an angle θ with the upward vertical the speed of P is v m s^{-1} (see diagram).

Remember:

$$KE = \frac{1}{2}mv^2 \quad PE = mgh$$

(i) Show that $v^2 = 33.32 - 15.68 \cos \theta$.

[4]

Initially :

$$KE = \frac{1}{2} \times 0.03 \times 4.2^2 = 0.2646$$

$$PE = 0$$

$$\text{Total energy} = 0.2646$$

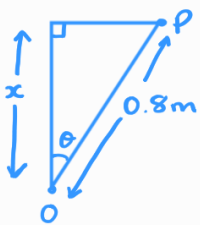
At angle θ :

$$KE = \frac{1}{2} \times 0.03 \times v^2 = 0.015v^2 \quad \cos \theta = \frac{x}{0.8}$$

$$PE = -0.03g(0.8 - 0.8 \cos \theta)$$

$$= 0.2352(\cos \theta - 1)$$

$$\text{Total energy} = 0.015v^2 + 0.2352(\cos \theta - 1)$$



By conservation of energy:

$$0.2646 = 0.2352(\cos \theta - 1) + 0.015v^2$$

$$17.64 = 15.68 \cos \theta - 15.68 + v^2$$

$$v^2 = 33.32 - 15.68 \cos \theta \quad (\text{as required})$$

(ii) Prove that the bead is never at rest.

[1]

(iii) Find the maximum value of v .

[2]

$$\text{ii. If } v = 0, \Rightarrow 0 = 33.32 - 15.68 \cos \theta$$
$$\cos \theta = 2.125$$

2.125 > 1 so this has no solutions, it can never be at rest.

iii. At max when $\cos \theta = -1$:

$$v^2 = 33.32 - 15.68 \times -1$$

$$v^2 = 49$$

$$v = 7 \text{ ms}^{-1}$$

3 (i) Write down the dimension of density.

[1]

The workings of an oil pump consist of a right, solid cylinder which is partially submerged in oil. The cylinder is free to oscillate along its central axis which is vertical. If the base area of the pump is 0.4 m^2 and the density of the oil is 920 kg m^{-3} then the period of oscillation of the pump is 0.7 s .

A student assumes that the period of oscillation of the pump is dependent only on the density of the oil, ρ , the acceleration due to gravity, g , and the surface area, A , of the circular base of the pump. The student attempts to test this assumption by stating that the period of oscillation, T , is given by $T = C\rho^\alpha g^\beta A^\gamma$ where C is a dimensionless constant.

(ii) Use dimensional analysis to find the values of α , β and γ .

[4]

$$\text{i. } \rho = \text{M L}^{-3}$$

$$\text{ii. } T = C\rho^\alpha g^\beta A^\gamma$$
$$T = (\text{M L}^{-3})^\alpha (\text{L T}^{-2})^\beta (\text{L}^2)^\gamma$$
$$T = \text{M}^\alpha \text{L}^{-3\alpha + \beta + 2\gamma} \text{T}^{-2\beta}$$

$$M: \alpha = 0$$

$$T: 1 = -2\beta$$

$$\beta = -1/2$$

$$L: -3\alpha + \beta + 2\gamma = 0$$

$$0 - 1/2 + 2\gamma = 0$$

$$2\gamma = 1/2$$

$$\gamma = 1/4$$

$$\therefore \alpha = 0, \beta = -1/2, \gamma = 1/4$$

(iii) Hence give the value of C to 3 significant figures.

[2]

(iv) Comment, with justification, on the assumption made by the student that the formula for the period of oscillation of the pump was dependent on only ρ , g and A .

[2]

$$\text{iii. } T = C g^{-1/2} A^{1/4}$$

sub in known values: $T = 0.7$ at $A = 0.4$

$$0.7 = C g^{-1/2} A^{1/4}$$

$$0.7 = 0.254C$$

$$C = 2.76$$

iv. $\alpha = 0$ so period of oscillation is not dependant on density in this model.

- 4 A car of mass 1250 kg experiences a resistance to its motion of magnitude kv^2 N, where k is a constant and v m s⁻¹ is the car's speed. The car travels in a straight line along a horizontal road with its engine working at a constant rate of P W. At a point A on the road the car's speed is 15 m s⁻¹ and it has an acceleration of magnitude 0.54 m s⁻². At a point B on the road the car's speed is 20 m s⁻¹ and it has an acceleration of magnitude 0.3 m s⁻².

(i) Find the values of k and P .

[7]

$$kv^2 \longleftarrow O \longrightarrow f$$

$$P = f v \quad \Rightarrow \quad f = \frac{P}{v}$$

$$\text{At } A : f - kv^2 = ma$$

$$\frac{P}{15} - k \times 15^2 = 1250 \times 0.54$$

$$\frac{P}{15} - 225k = 675$$

$$P - 3375k = 10125$$

$$\text{At } B : f - kv^2 = ma$$

$$\frac{P}{20} - k \times 20^2 = 1250 \times 0.3$$

$$P - 8000k = 7500$$

Setting these equations for P equal to each other:

$$10125 + 3375k = 7500 + 8000k$$

$$4625k = 2625$$

$$\underline{k = 0.568} \quad (3\text{sf})$$

$$P = 7500 + 8000 \times 0.568 \dots$$

$$P = 12040.54054 \text{ W}$$

$$\underline{P = 12041 \text{ W}}$$

The power is increased to 15 kW.

(ii) Calculate the maximum steady speed of the car on a straight horizontal road.

[3]

Max speed happens when acceleration = 0

$$F - kv^2 = 0$$

$$\frac{15000}{v} = 0.568 v^2$$

$$v^3 = 26428.6$$

$$v = 29.8 \text{ ms}^{-1} \text{ (3sf)}$$

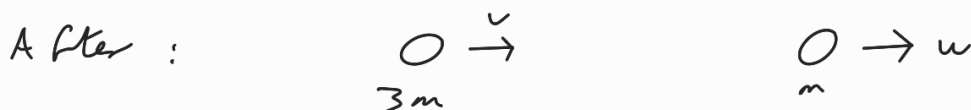
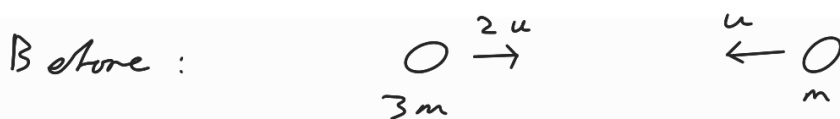
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The masses of two spheres A and B are $3m \text{ kg}$ and $m \text{ kg}$ respectively. The spheres are moving towards each other with constant speeds $2u \text{ ms}^{-1}$ and $u \text{ ms}^{-1}$ respectively along the same straight line towards each other on a smooth horizontal surface (see diagram). The two spheres collide and the coefficient of restitution between the spheres is e . After colliding, A and B both move in the same direction with speeds $v \text{ ms}^{-1}$ and $w \text{ ms}^{-1}$, respectively.

(i) Find an expression for v in terms of e and u .

[6]



$$e = \frac{w - v}{2u - (-u)}$$

$$e = \frac{w - v}{3u} \Rightarrow w = 3ue + v$$

$$\begin{aligned}
 \text{Momentum: } 3m(2u) + m(-u) &= 3m(v) + m(w) \\
 6mu - mu &= 3mv + mw \\
 5u &= 3v + w \\
 5u &= 3v + 3ue + v \\
 4v &= -3ue + 5u \\
 v &= \frac{u}{4}(5 - 3e)
 \end{aligned}$$

(ii) Write down unsimplified expressions in terms of e and u for

(a) the total kinetic energy of the spheres before the collision, [1]

(b) the total kinetic energy of the spheres after the collision. [2]

$$a. \text{ before} = \frac{1}{2}(3m)(2u)^2 + \frac{1}{2}(m)(-u)^2$$

$$b. \text{ after} = \frac{1}{2}(3m)\left(\frac{u}{4}\right)^2(5-3e)^2 + \frac{1}{2}(m)\left(\frac{u}{4}\right)^2(5+9e)^2$$

(iii) Given that the total kinetic energy of the spheres after the collision is λ times the total kinetic energy before the collision, show that

$$\lambda = \frac{27e^2 + 25}{52}.$$

[3]

$$\frac{13}{2} \cancel{mu^2} \times \lambda = \frac{3}{32} \cancel{mu^2} (5-3e)^2 + \frac{1}{32} \cancel{mu^2} (5+9e)^2$$

$$208\lambda = 3(25 - 30e + 9e^2) + (25 + 90e + 81e^2)$$

$$208\lambda = 100 + 108e^2$$

$$\lambda = \frac{27e^2 + 25}{52}$$

(iv) Comment on the cases when

(a) $\lambda = 1$,

(b) $\lambda = \frac{25}{52}$.

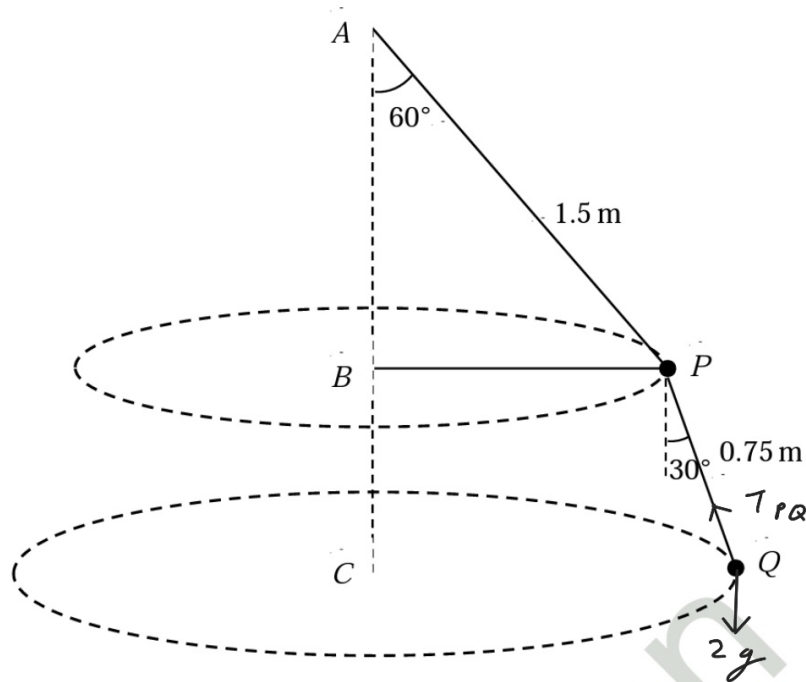
[3]

$$a. \lambda = 1 \Rightarrow 52 = 27e^2 + 25$$
$$e = 1$$

$e = 1$ is a perfectly elastic collision, meaning no energy is lost.

$$b. \lambda = \frac{25}{52} \Rightarrow e = 0$$

The collision is perfectly inelastic, the particles join together.



The fixed points A , B and C are in a vertical line with A above B and B above C . A particle P of mass 2.5 kg is joined to A , to B and to a particle Q of mass 2 kg, by three light rods where the length of rod AP is 1.5 m and the length of rod PQ is 0.75 m. Particle P moves in a horizontal circle with centre B . Particle Q moves in a horizontal circle with centre C at the same constant angular speed ω as P , in such a way that A , B , P and Q are coplanar. The rod AP makes an angle of 60° with the downward vertical, rod PQ makes an angle of 30° with the downward vertical and rod BP is horizontal (see diagram).

(i) Find the tension in the rod PQ . [2]

(ii) Find ω . [3]

$$i. \text{ Resolve vertically : } T_{PQ} \cos 30 = 2g$$

$$T_{PQ} = 22.6 \text{ N}$$

$$ii. \quad r = 1.5 \sin 60 + 0.75 \sin 30$$

$$r = 1.674$$

$$T_{PQ} \sin 30 = 2r\omega^2$$

$$\omega^2 = \frac{22.6 \sin 30}{2 \times 1.674}$$

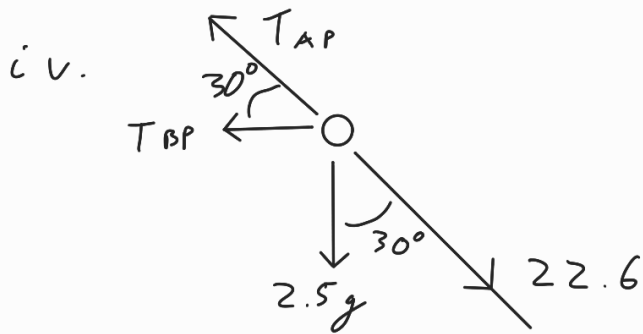
$$\omega^2 = 3.375$$

$$\omega = 1.84 \text{ rad s}^{-1}$$

(iii) Find the speed of P . [1]

(iv) Find the tension in the rod AP . [3]

$$\text{i i i. } v_p = 1.5 \sin 60 \times 1.84 \\ = 2.39 \text{ ms}^{-1}$$



Resolve vertically: $T_{AP} \sin 30 = 2.5g + 22.6 \cos 30$
 $\frac{1}{2} T_{AP} = 44.1$
 $T_{AP} = 88.2 \text{ N}$

(v) Hence find the magnitude of the force in rod BP .
Decide whether this rod is under tension or compression. [4]

Resolve horizontally: $T_{BP} + T_{AP} \cos 30 - 22.6 \sin 30 = 2.5 \times (1.5 \sin 60) \omega^2$
 $T_{BP} + 88.2 \cos 30 - 11.3 = 2.5 \times 1.299 \times 3.375$
 $T_{BP} = -54.1 \text{ N}$

$$\text{Magnitude} = 54.1$$

$-54.1 < 0$ so the rod BP is under compression.